Scaling Properties of Urban Radiance

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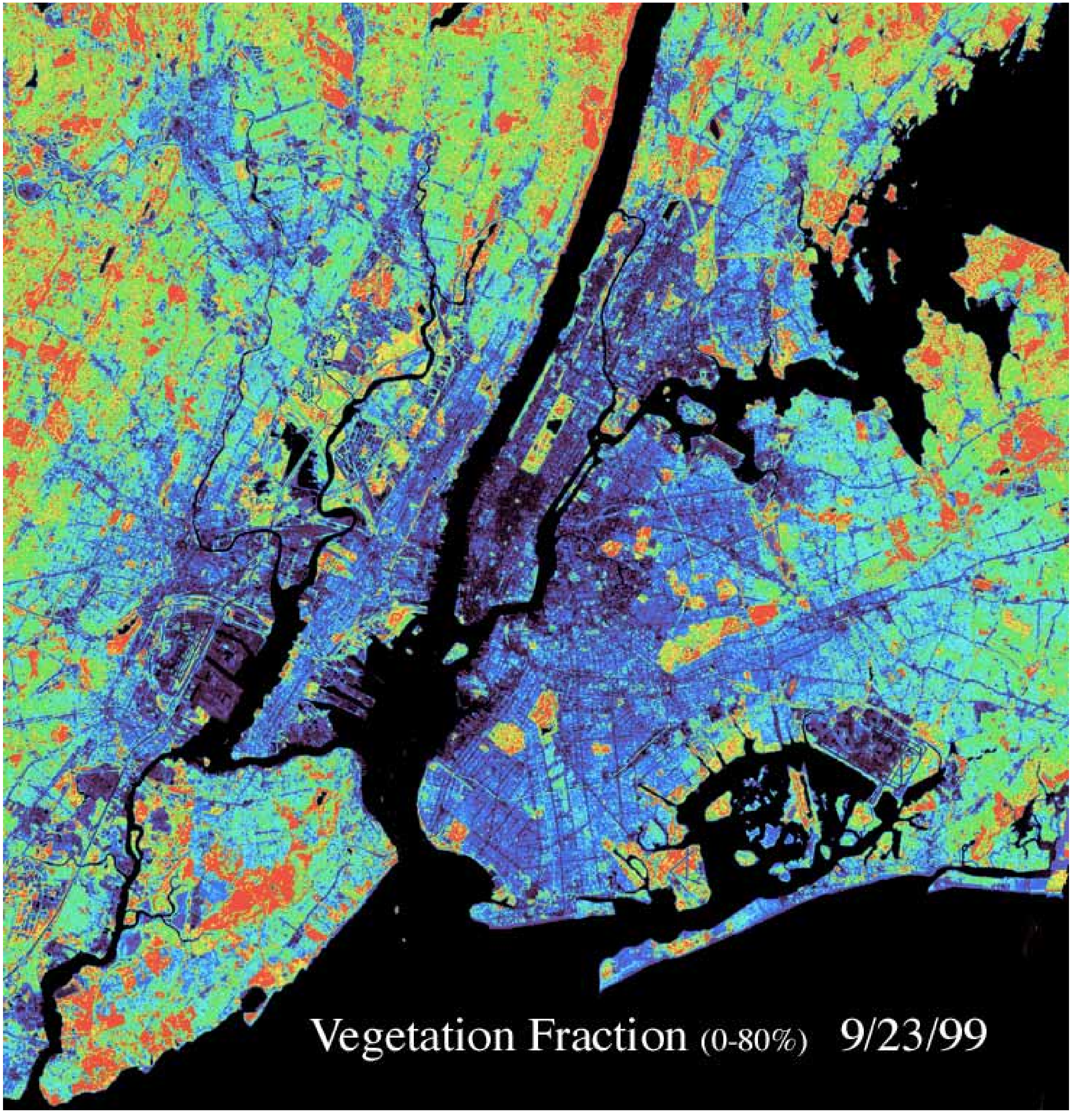
Research

Energy/Mass Flux Scaling
Urban Vegetation Abundance
Aerosol Distribution

Application

Urban Microclimate
Urban Ecology
Air Quality


Landsat 7 ETM+ (RGB=7/4/2) 9/23/99


Vegetation Fraction (0-80%) 9/23/99

A Heirarchy of Questions

Urban Vegetation Abundance & Spatial Distribution - *How does it affect energy consumption & air quality?*

Spectral Mixing & Fine Scale Vegetation Mapping - *How Linear? How Accurate?*

Scale Dependence & Spectral Dimensionality - *What is the tradeoff?*

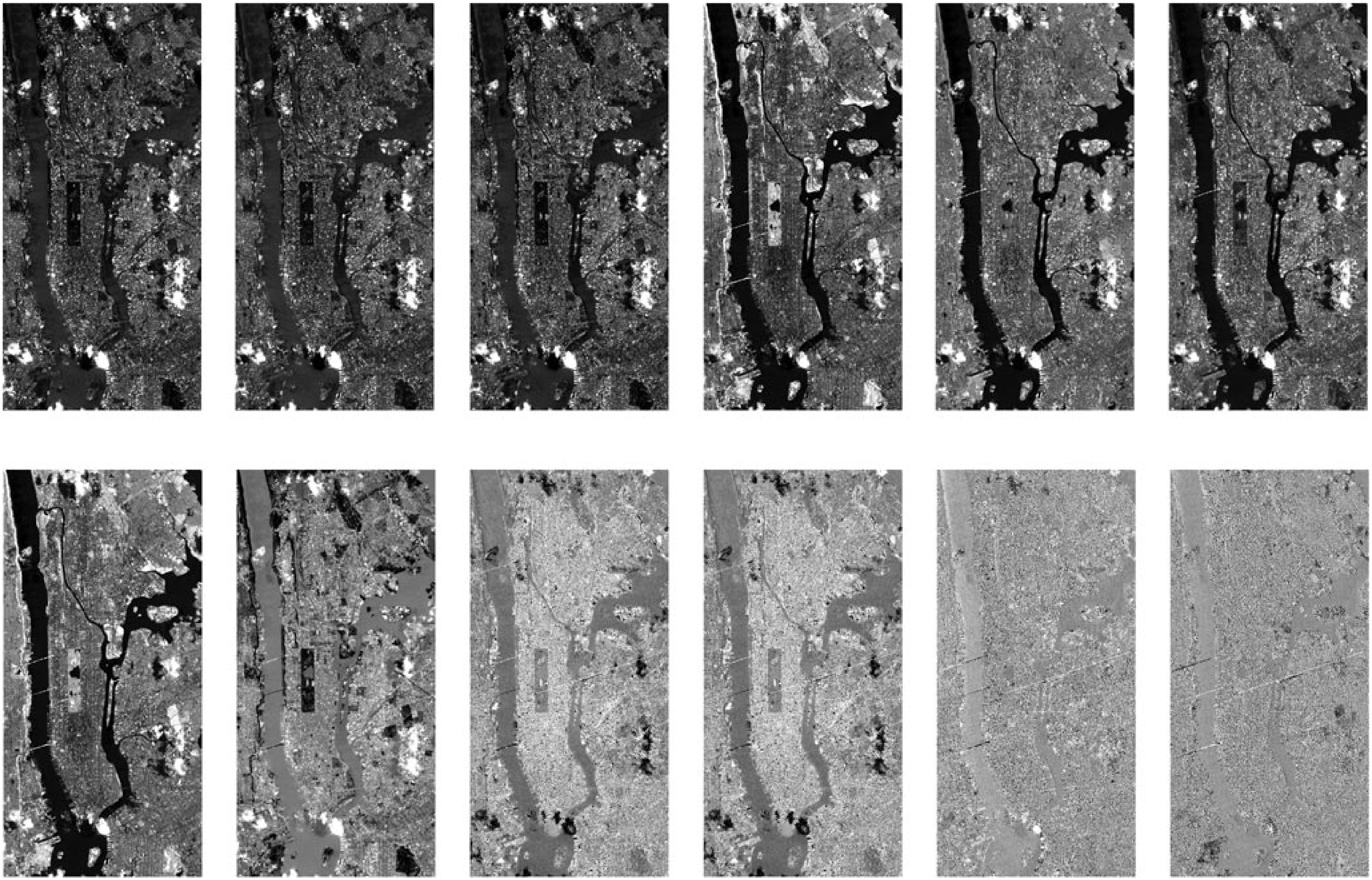
Signal/Noise & Spectral Resolution - *Effect on apparent spectral resolution?*

The questions will be addressed in an analysis of 7 U.S. Cities.

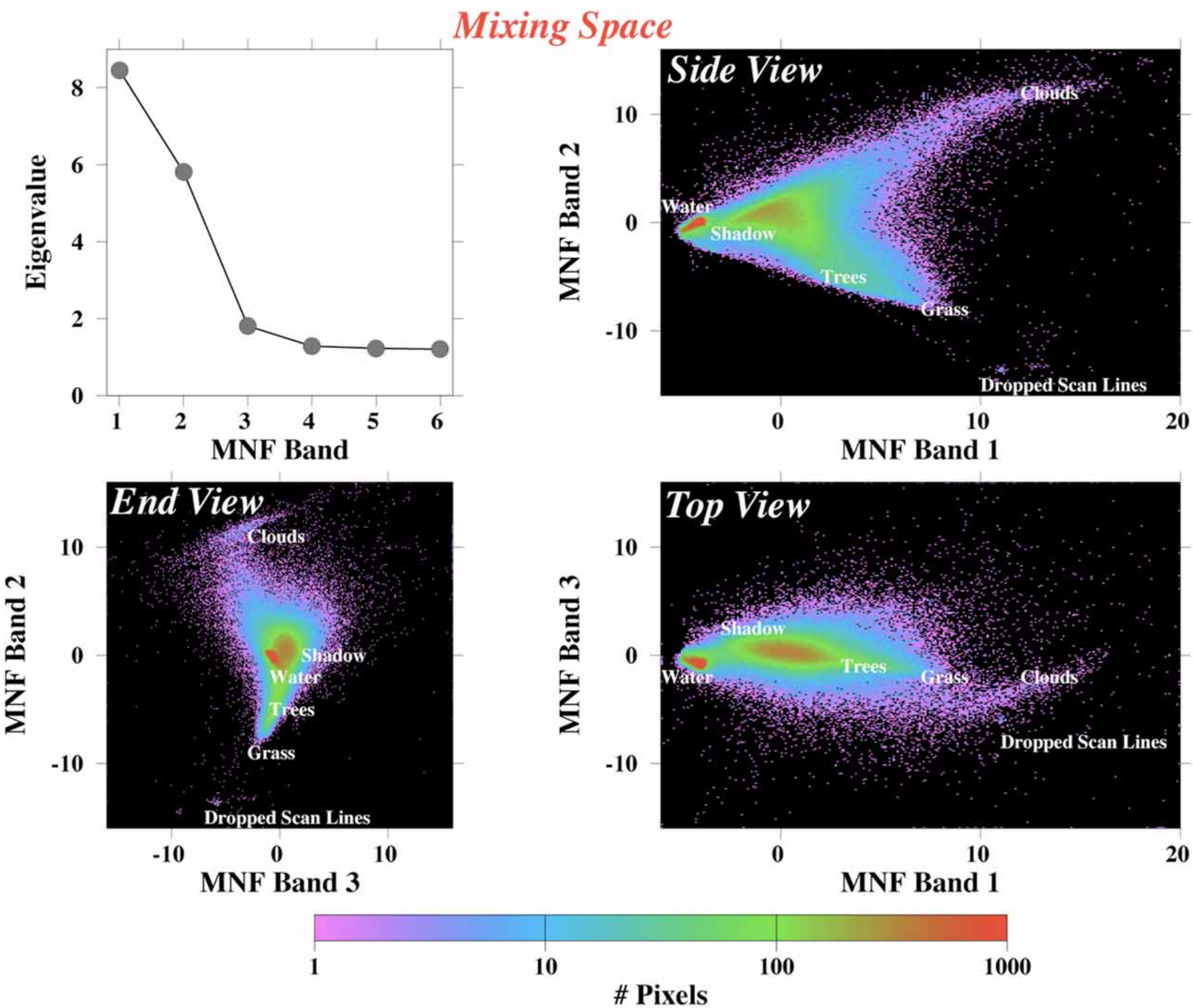
The objective is to quantify and analyse abundance and distribution of Urban & Suburban vegetation.

Atlanta, Chicago, Houston, Los Angeles, New York, Phoenix, Seattle.

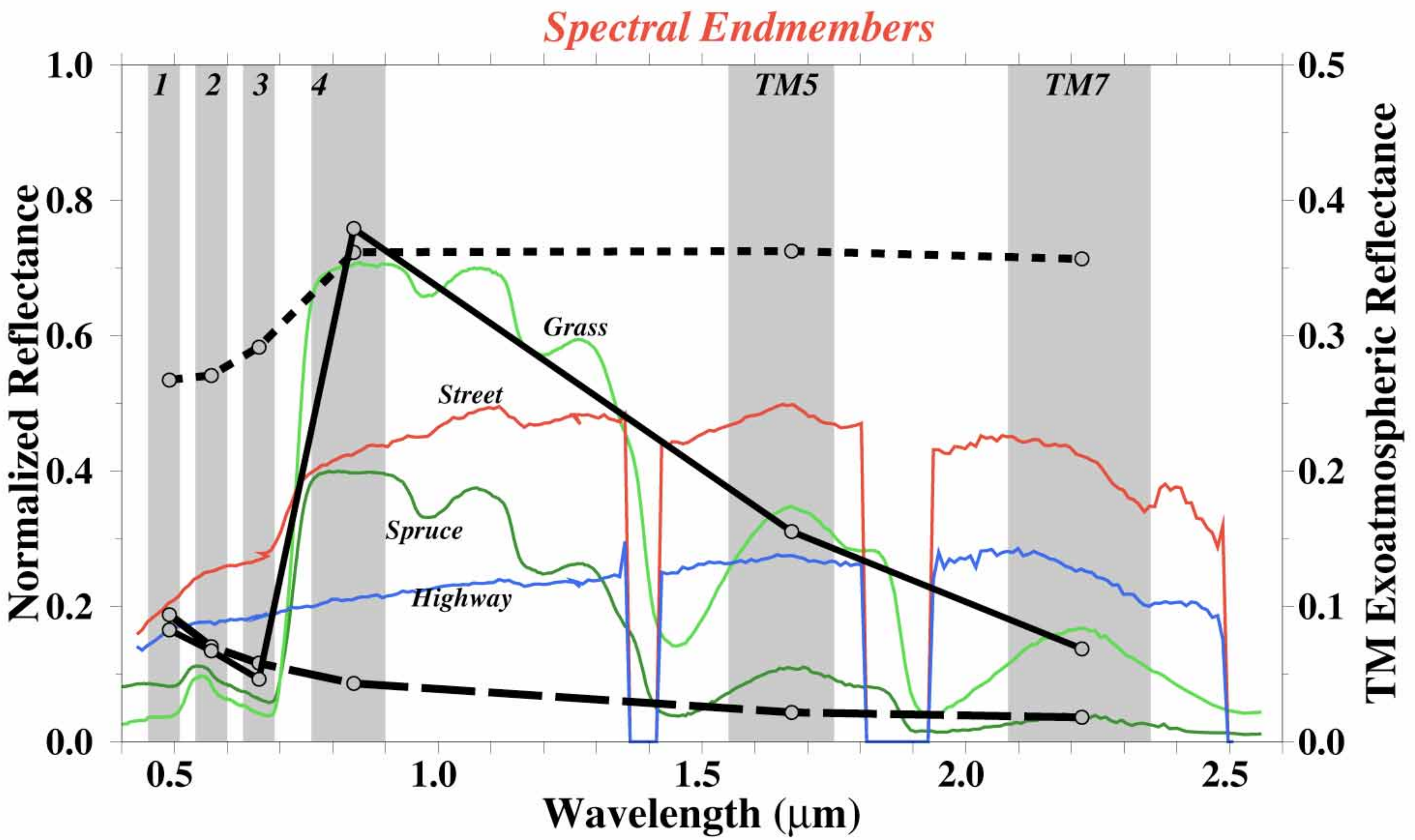
TM band 1 | TM band 2 | TM band 3 | TM band 4 | TM band 5 | TM band 7



MNF component 1 | MNF component 2 | MNF component 3 | MNF component 4 | MNF component 5 | MNF component 6



Mixing Space
Eigenvalue
MNF Band
Side View
MNF Band 2
MNF Band 1
Clouds
Water
Shadow
Trees
Grass
Dropped Scan Lines
End View
MNF Band 3
Top View
Pixels
1
10
100
1000



Spectral Endmembers
Normalized Reflectance
TM Exoatmospheric Reflectance
Wavelength (μm)
1
2
3
4
TM5
TM7
Grass
Street
Spruce
Highway
1.0
0.8
0.6
0.4
0.2
0.0
0.5
0.4
0.3
0.2
0.1
0.0
0.5
1.0
1.5
2.0
2.5

Spectral reflectance can be described as a linear combination of endmember spectra as:

$$f_1\mathbf{E}_1(\lambda) + f_2\mathbf{E}_2(\lambda) \ldots + f_n\mathbf{E}_n(\lambda) = \mathbf{R}(\lambda)$$

$\mathbf{R}(\lambda)$ is the observed reflectance profile, a continuous function of wavelength λ.
$\mathbf{E}_i(\lambda)$ are the endmember spectra and
f_i are the corresponding fractions of the n endmembers

Continuous reflectance profiles are represented as vectors of discrete reflectance estimates at specific wavelengths as:

$$\mathbf{E}(\lambda) = [e_{\lambda 1}, e_{\lambda 2} \ldots e_{\lambda n}] \quad \text{and} \quad \mathbf{R}(\lambda) = [r_{\lambda 1}, r_{\lambda 2} \ldots r_{\lambda n}]$$

$r_{\lambda i}$ represents a portion of the observed reflectance spectrum
$\mathbf{R}(\lambda)$, integrated over a finite spectral band with a center wavelength λ_i and
$e_{\lambda i}$ represents observed reflectance from the corresponding endmember $\mathbf{E}(\lambda)$.

The continuous linear mixing model can be represented in discrete form as a system of linear mixing equations

$$f_j\mathbf{e}_{ij} = \mathbf{r}_i \qquad i = 1,b \quad \text{and} \quad j = 1,n$$

The system of b linear equations can be written as:

$$\mathbf{Ef} = \mathbf{r}$$

The overdetermined linear mixing model, incorporating measurement error:

$$\mathbf{r} = \mathbf{Ef} + \varepsilon$$

ε is an error vector which must be minimized to find the fraction vector $\mathbf{f}$ which gives the best fit to the observed reflectance vector $\mathbf{r}$. Since $\varepsilon = \mathbf{r} - \mathbf{Ef}$, we seek to minimize:

$$\varepsilon^T\varepsilon = (\mathbf{r} - \mathbf{Ef})(\mathbf{r} - \mathbf{Ef}).$$

In the case of uncorrelated noise, the well known least squares solution is given by:

$$\mathbf{f} = (\mathbf{E}^T \mathbf{E})^{-1} \mathbf{E}^T \mathbf{r}$$

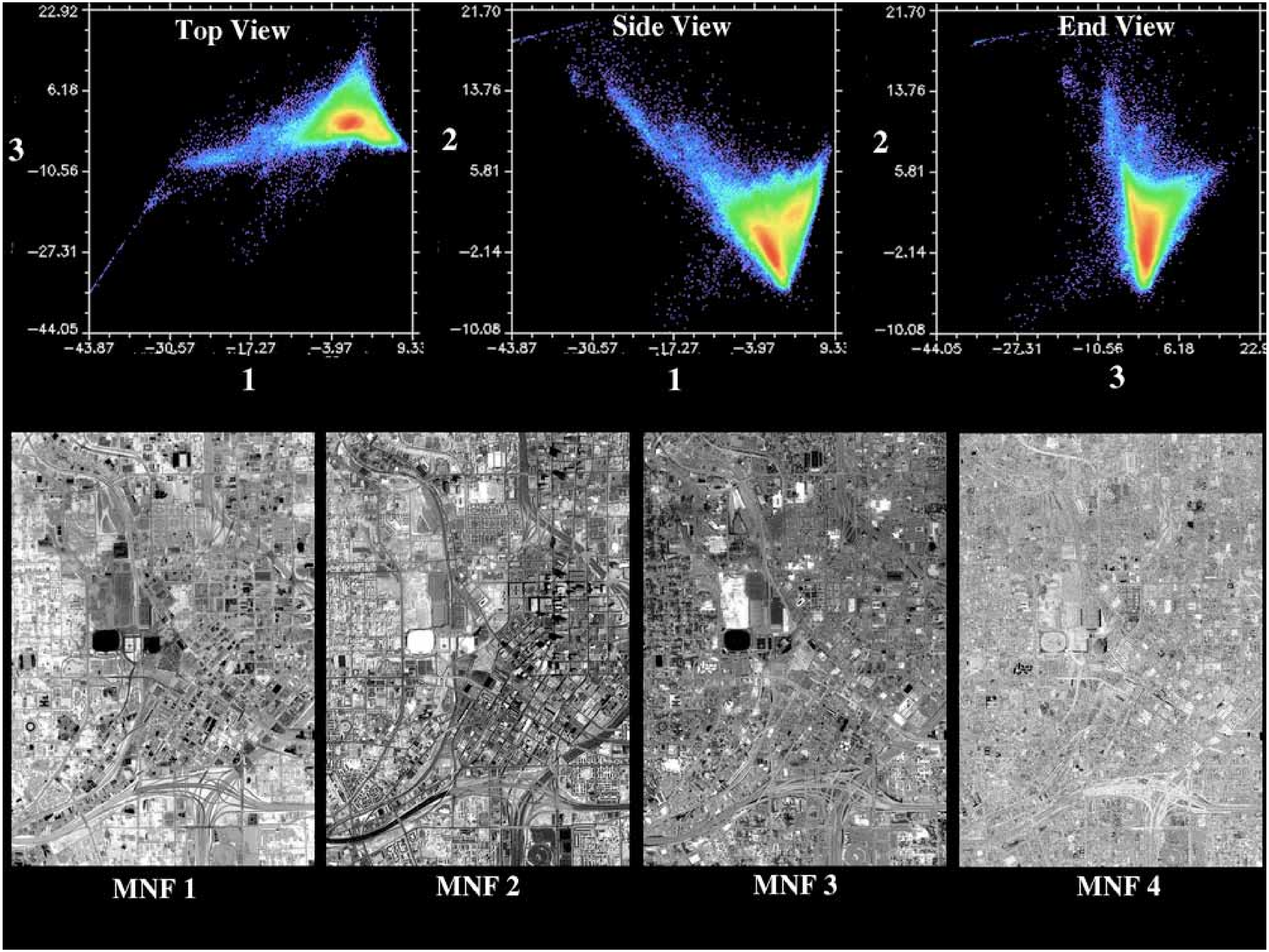
3 Endmember Linear Mixing Model

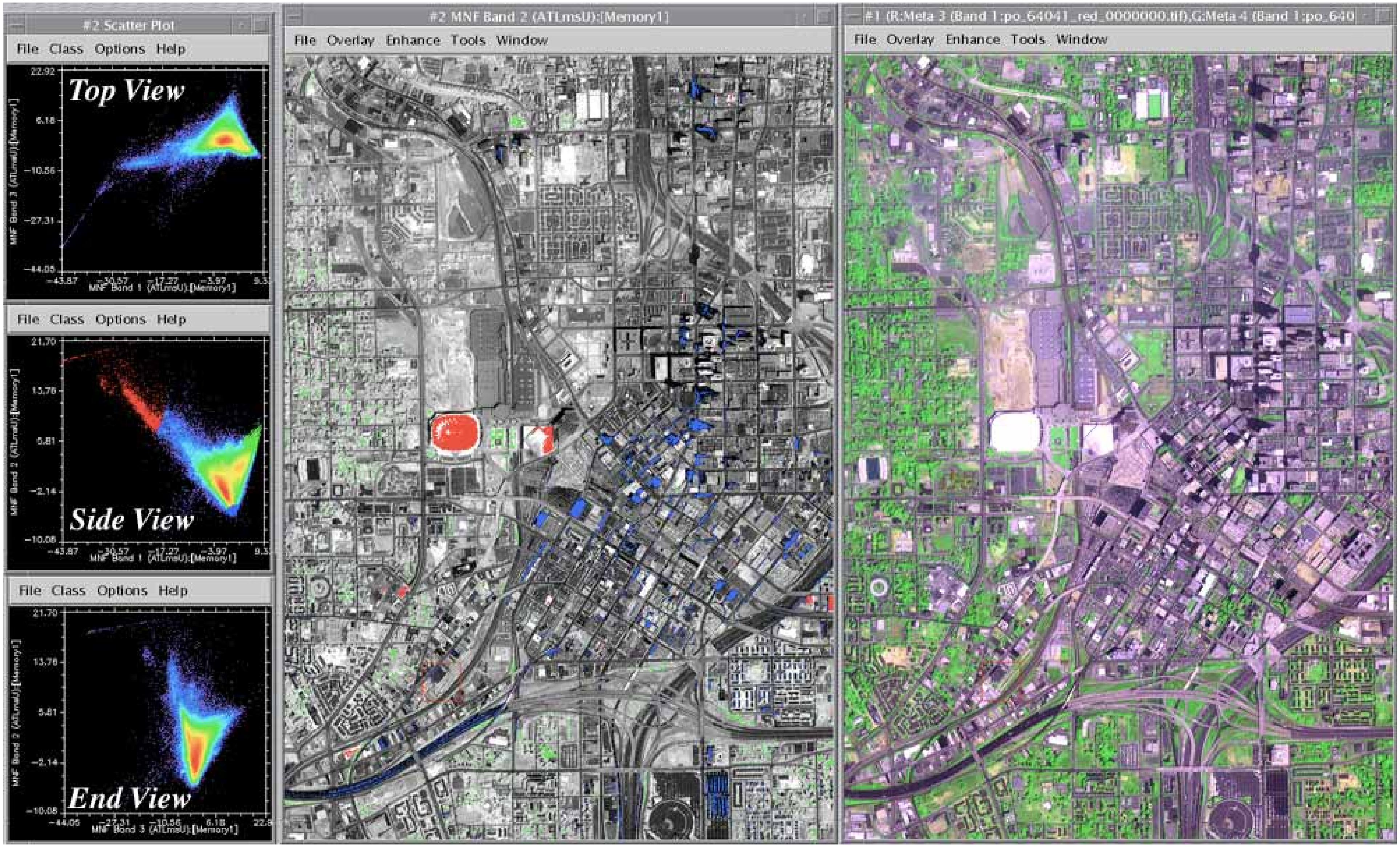
$$
\begin{array}{ccccccc}
f_1\mathbf{E}_{1\lambda1} & & f_2\mathbf{E}_{2\lambda1} & & f_3\mathbf{E}_{3\lambda1} & & R_{\lambda1} \\
f_1\mathbf{E}_{1\lambda2} & & f_2\mathbf{E}_{2\lambda2} & & f_3\mathbf{E}_{3\lambda2} & & R_{\lambda2} \\
f_1\mathbf{E}_{1\lambda3} & + & f_2\mathbf{E}_{2\lambda3} & + & f_3\mathbf{E}_{3\lambda3} & = & R_{\lambda1} \\
f_1\mathbf{E}_{1\lambda4} & & f_2\mathbf{E}_{2\lambda4} & & f_3\mathbf{E}_{3\lambda4} & & R_{\lambda4} \\
f_1\mathbf{E}_{1\lambda5} & & f_2\mathbf{E}_{2\lambda5} & & f_3\mathbf{E}_{3\lambda5} & & R_{\lambda5} \\
f_1\mathbf{E}_{1\lambda6} & & f_2\mathbf{E}_{2\lambda6} & & f_3\mathbf{E}_{3\lambda6} & & R_{\lambda6}
\end{array}
$$

To first order, radiances mix linearly in proportion to area.

Given some knowledge of the spectral endmembers (E), it is possible to* estimate *fractions (f) contributing to a spectrally mixed radiance measurement (R).

~~SubPixel Resolution~~


Top View
Side View
End View
22.92
6.18
-10.56
-27.31
-44.05
21.70
13.76
5.81
-2.14
-10.08
-43.87
-30.57
-17.27
-3.97
-44.05
-27.31
-10.56
6.18
1
2
3
MNF 1
MNF 2
MNF 3
MNF 4

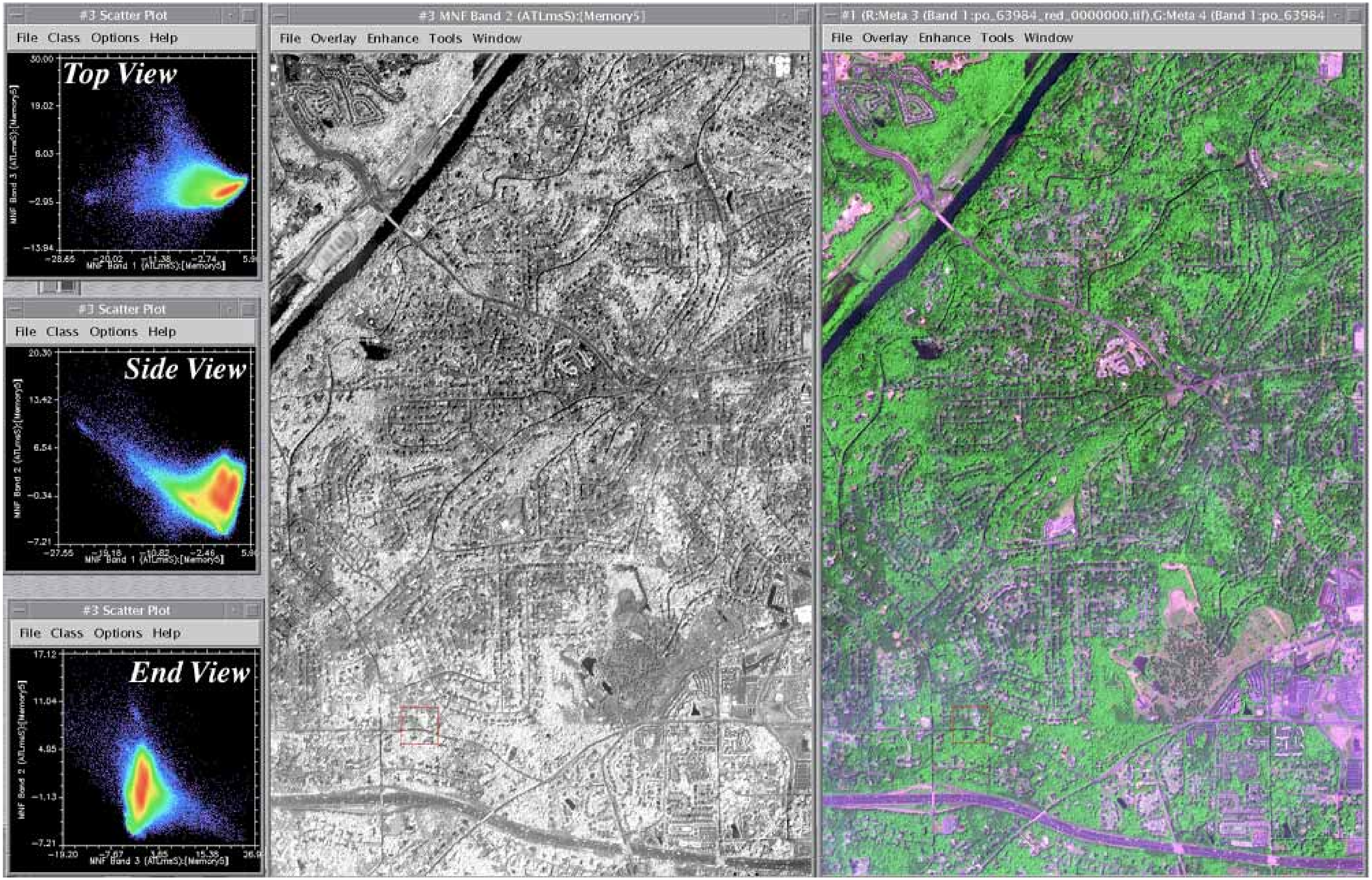


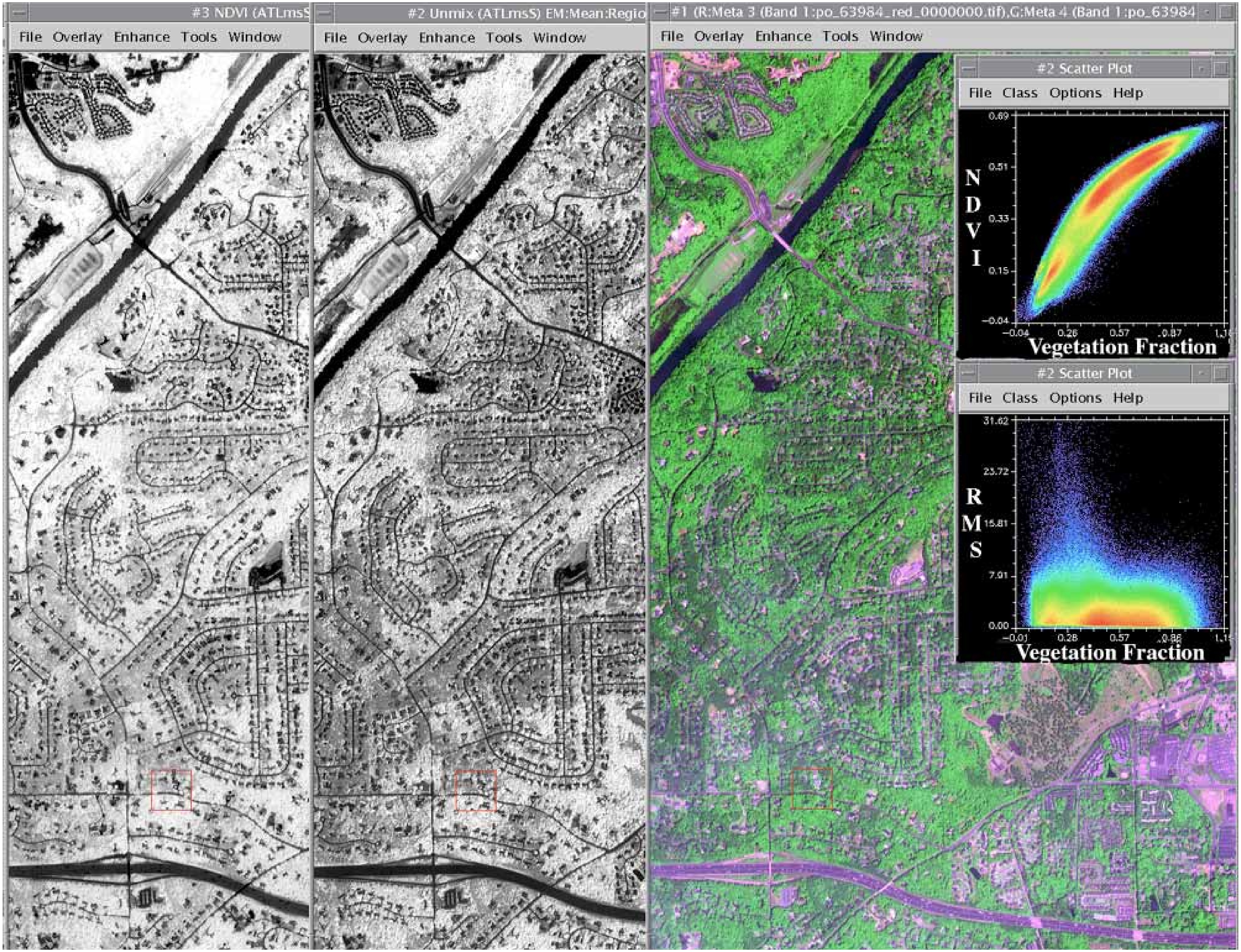
#2 Scatter Plot
File Class Options Help
Top View
Side View
End View
MNF Band 1 (ATLmsU):[Memory1]
MNF Band 2 (ATLmsU):[Memory1]
MNF Band 3 (ATLmsU):[Memory1]
#2 MNF Band 2 (ATLmsU):[Memory1]
File Overlay Enhance Tools Window
#1 (R:Meta 3 (Band 1:po_64041_red_0000000.tif),G:Meta 4 (Band 1:po_640

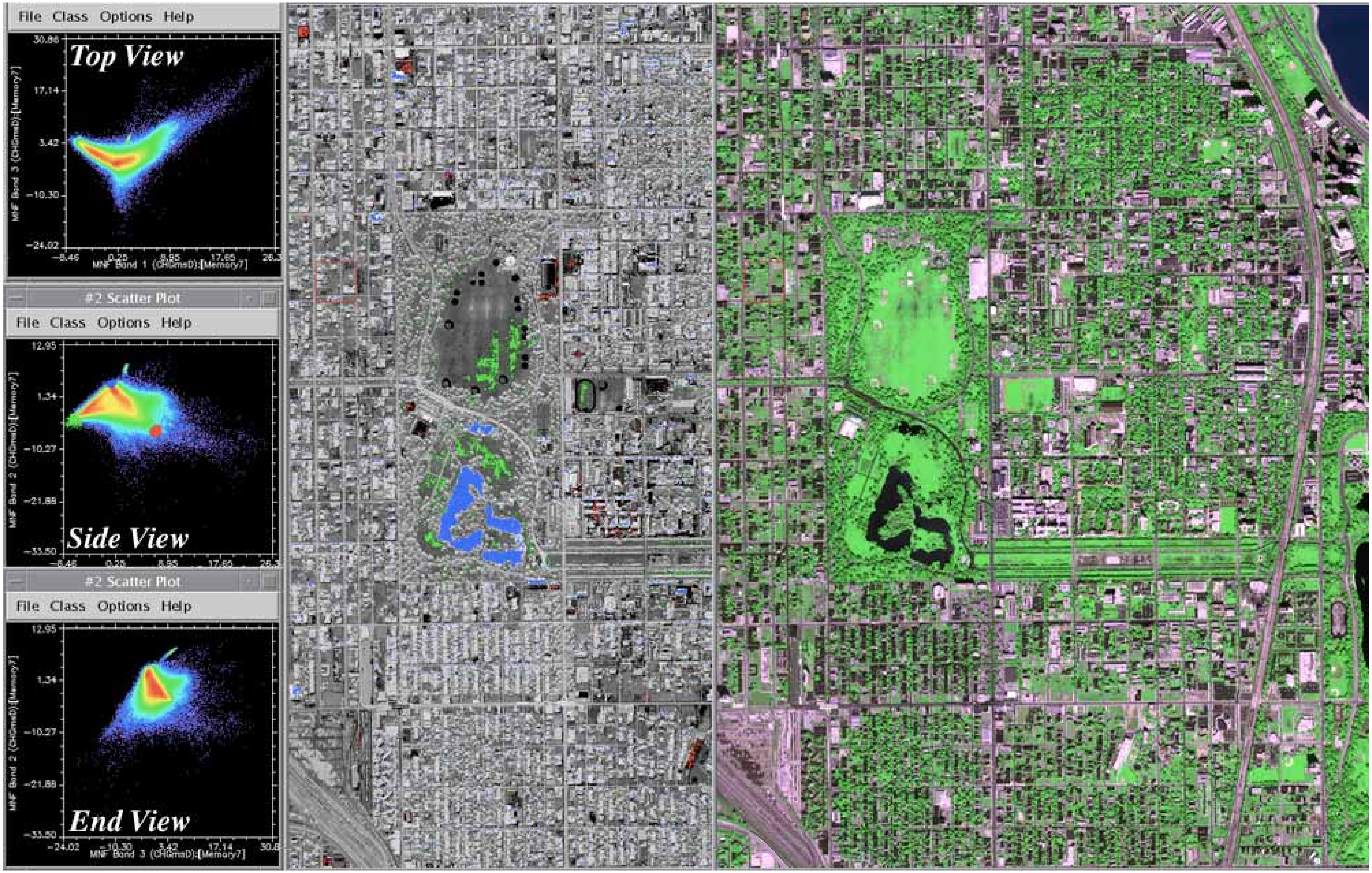
Atlanta GA, 5/18/2000

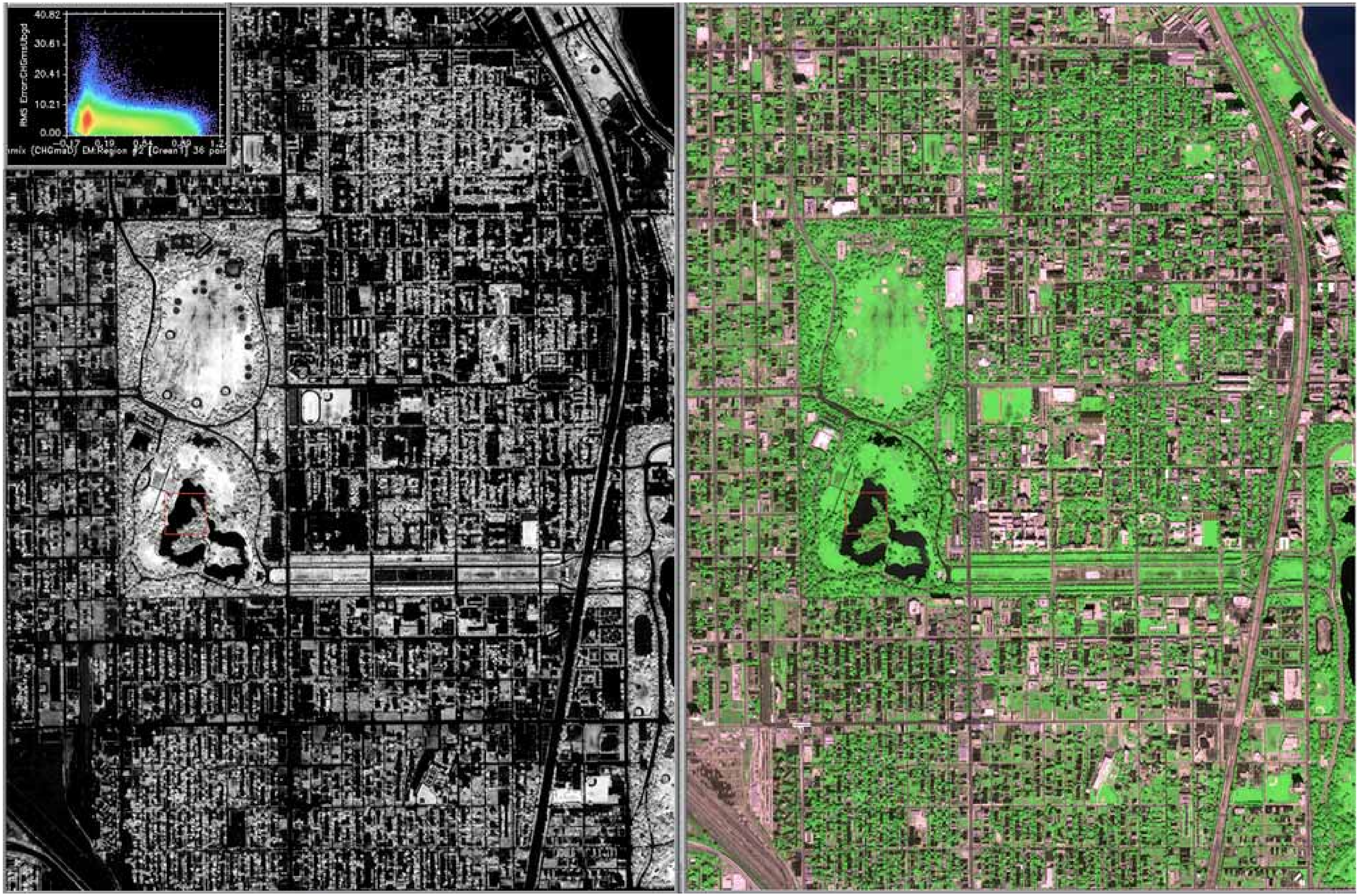
Ikonos (RGB=341)

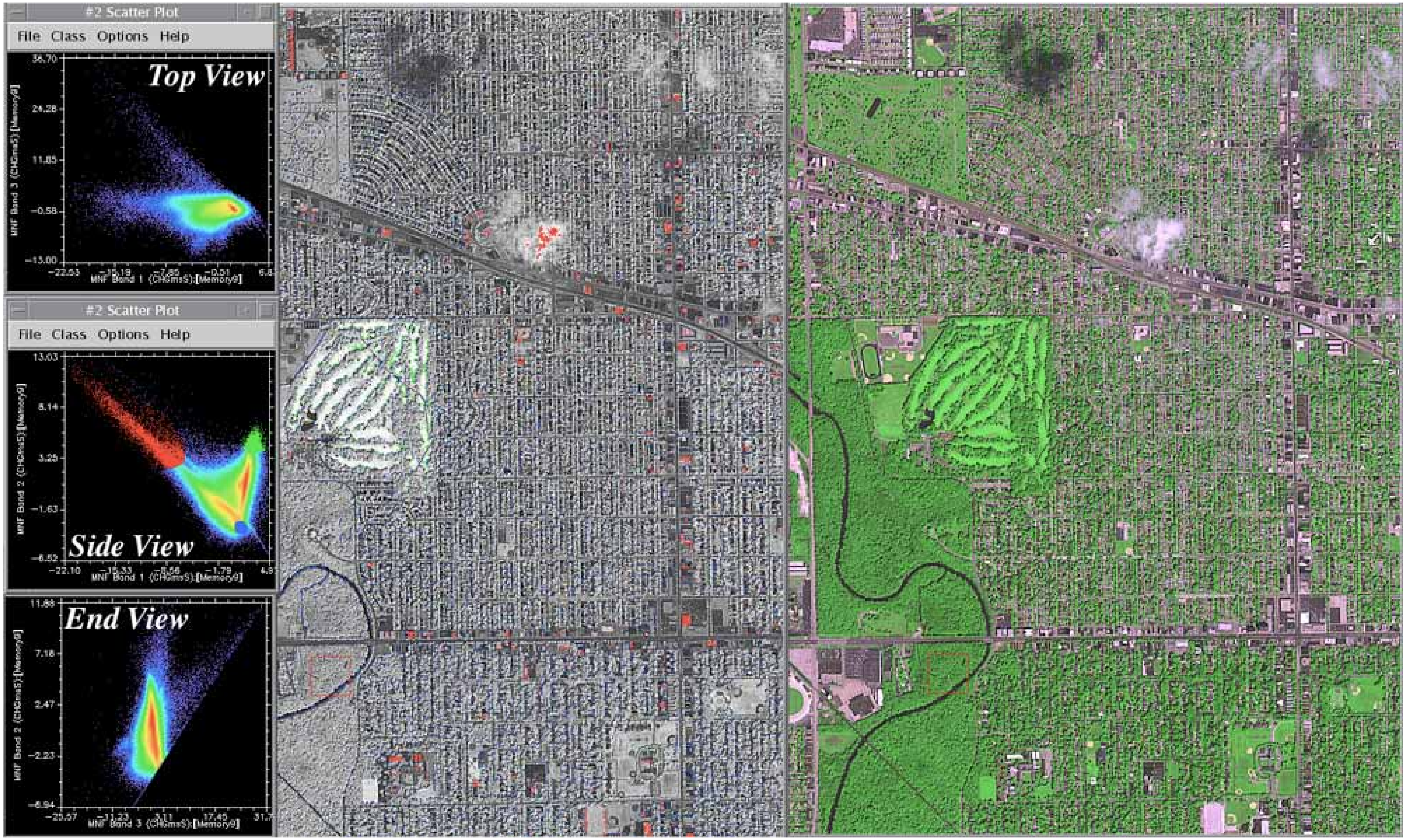
Vegetation Fraction


#3 Scatter Plot
File Class Options Help
Top View
30.00
19.02
8.03
−2.95
−13.94
−28.65
−20.02
−11.38
−2.74
MNF Band 3 (ATLmsS):[Memory5]
MNF Band 1 (ATLmsS):[Memory5]
#3 Scatter Plot
File Class Options Help
Side View
20.30
13.42
6.54
−0.34
−7.21
−27.55
−19.18
−10.82
−2.46
MNF Band 2 (ATLmsS):[Memory5]
MNF Band 1 (ATLmsS):[Memory5]
#3 Scatter Plot
File Class Options Help
End View
17.12
11.04
4.95
−1.13
−7.21
−19.20
−7.67
3.85
15.38
MNF Band 2 (ATLmsS):[Memory5]
MNF Band 3 (ATLmsS):[Memory5]
#3 MNF Band 2 (ATLmsS):[Memory5]
File Overlay Enhance Tools Window
#1 (R:Meta 3 (Band 1:po_63984_red_0000000.tif),G:Meta 4 (Band 1:po_63984
File Overlay Enhance Tools Window

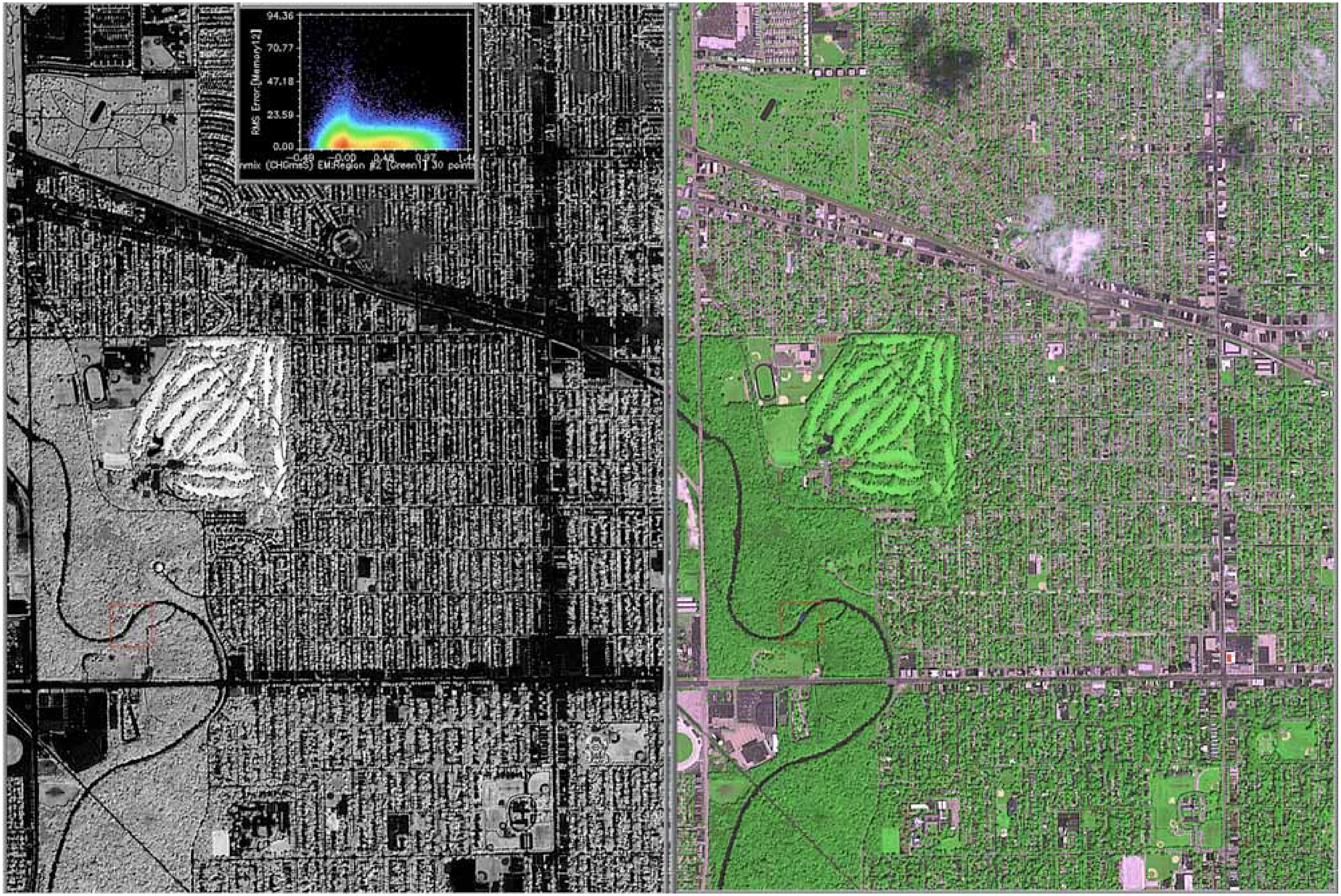

#3 NDVI (ATLmsS
File Overlay Enhance Tools Window
#2 Unmix (ATLmsS) EM:Mean:Regio
File Overlay Enhance Tools Window
#1 (R:Meta 3 (Band 1:po_63984_red_0000000.tif),G:Meta 4 (Band 1:po_63984
File Overlay Enhance Tools Window
#2 Scatter Plot
File Class Options Help
NDVI
Vegetation Fraction
#2 Scatter Plot
File Class Options Help
RMS
Vegetation Fraction

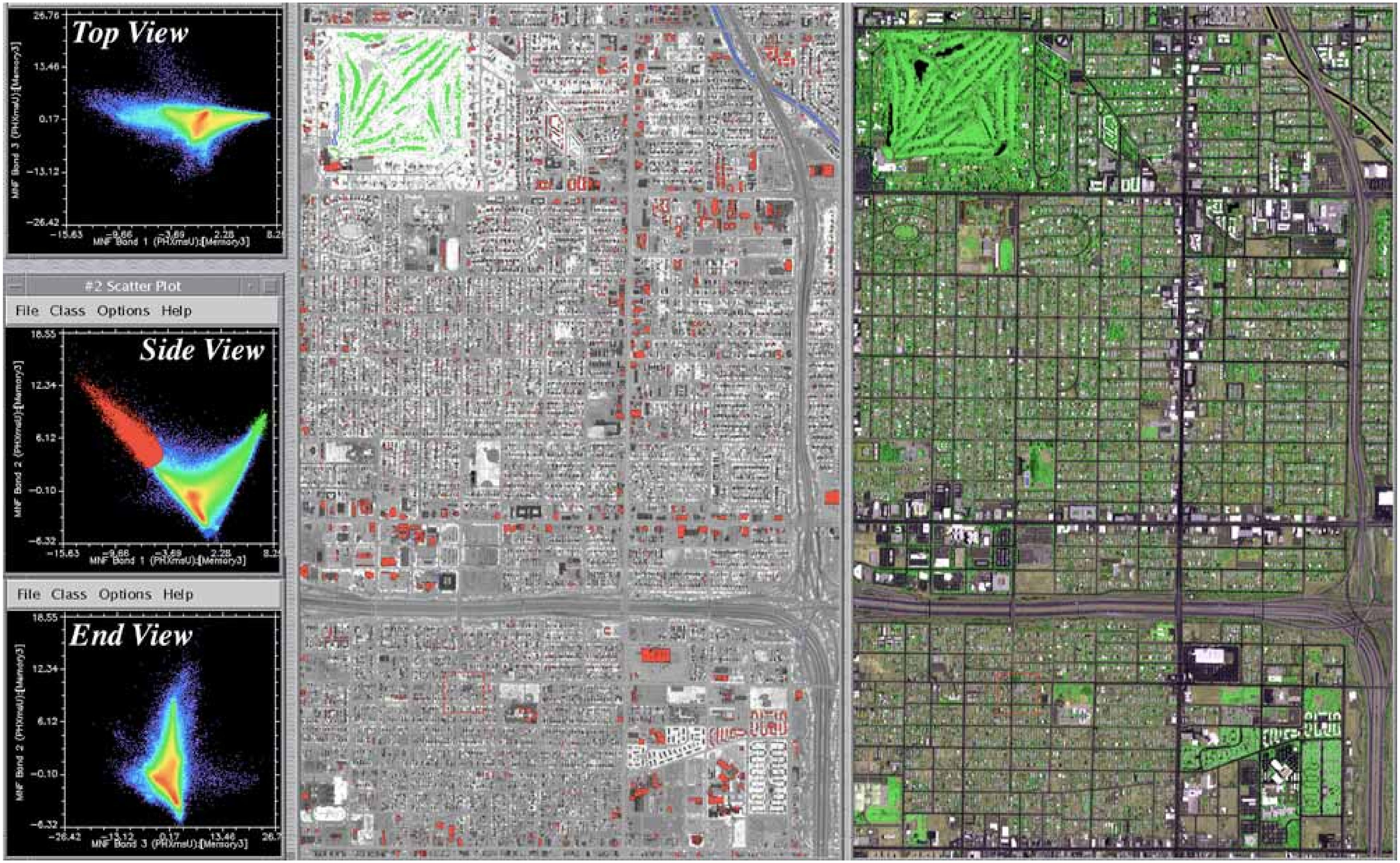

File Class Options Help
Top View
#2 Scatter Plot
File Class Options Help
Side View
#2 Scatter Plot
File Class Options Help
End View

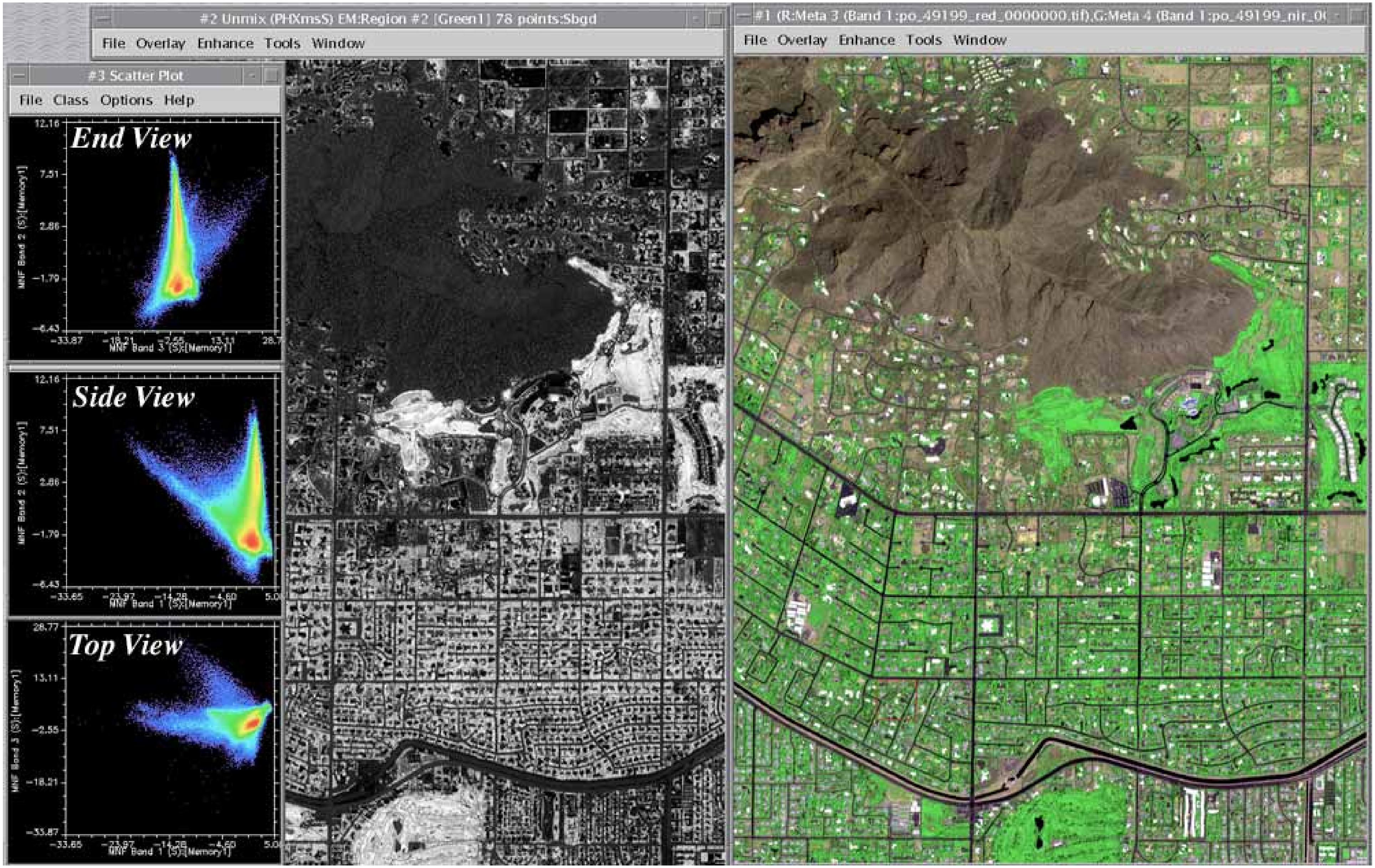

40.82
30.61
20.41
10.21
0.00
RMS Error/CHGmsUbgd
-0.17
0.19
0.54
0.89
mix (CHGmsU) EM:Region #2 [Green1] 36 poin

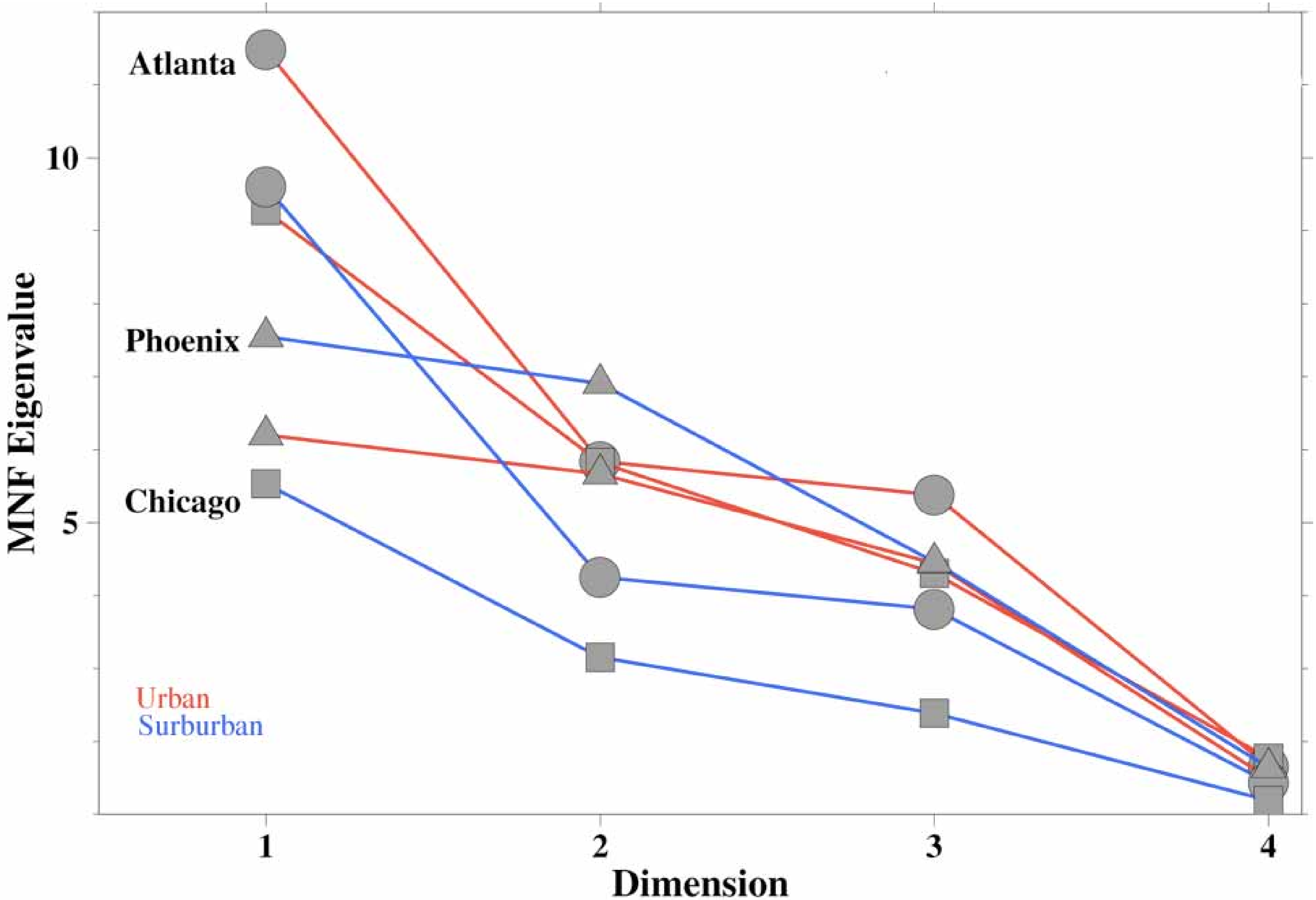


#2 Scatter Plot
File Class Options Help
Top View
MNF Band 3 (CHGmsS):[Memory9]
MNF Band 1 (CHGmsS):[Memory9]
#2 Scatter Plot
File Class Options Help
Side View
MNF Band 2 (CHGmsS):[Memory9]
MNF Band 1 (CHGmsS):[Memory9]
End View
MNF Band 2 (CHGmsS):[Memory9]
MNF Band 3 (CHGmsS):[Memory9]


94.36
70.77
47.18
23.59
0.00
RMS Error [Memory12]


Top View
MNF Band 3 (PHXmsU):[Memory3]
MNF Band 1 (PHXmsU):[Memory3]
#2 Scatter Plot
File Class Options Help
Side View
MNF Band 2 (PHXmsU):[Memory3]
MNF Band 1 (PHXmsU):[Memory3]
File Class Options Help
End View
MNF Band 2 (PHXmsU):[Memory3]
MNF Band 3 (PHXmsU):[Memory3]


#2 Unmix (PHXmsS) EM:Region #2 [Green1] 78 points:Sbgd
File Overlay Enhance Tools Window
#3 Scatter Plot
File Class Options Help
End View
Side View
Top View
#1 (R:Meta 3 (Band 1:po_49199_red_0000000.tif),G:Meta 4 (Band 1:po_49199_nir_0(
File Overlay Enhance Tools Window



Atlanta
Phoenix
Chicago
10
5
MNF Eigenvalue
1
2
3
4
Dimension
Urban
Surburban

Conclusions

Apparent dimensionality of Ikonos MSI imagery exceeds noise threshold.

Urban & Suburban areas analysed show3+ dimensional mixing spaces with significant nonlinearity.

The 3 endmember linear mixing model produces < 5% RMS misfit for vegetation component.

Urban & Suburban areas have similar eigenvalue distributions but distinct mixing spaces.